

Interesting Saddles
Lecture 2
Quantum Glass

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Cugliandolo, Grepel, da Silva Santos

Recap from last time: Starting from a Hamiltonian
 $H = \sum_{i_1 \dots i_p} J_{i_1 \dots i_p} \sigma_{i_1} \dots \sigma_{i_p}$ w/ $\sum_i \sigma_i^2 = N$

& $P(J_{i_1 \dots i_p}) \propto \exp \left\{ - \frac{J_{i_1 \dots i_p}^2}{2} \frac{2N^{p-1}}{p!} \right\}$

Leads to saddle point equations:

$\frac{\beta^2}{2} P Q_{ab}^{p-1} + (Q^{-1})_{ab} = 0$ $Q_{ab} = \frac{1}{N} \sum_{i=1}^N \langle \sigma_i^a \sigma_i^b \rangle$ $a=1, \dots, N$

High T: $Q_{ab} = \begin{pmatrix} 1 & & \\ & \ddots & \\ 0 & & 0 \end{pmatrix} \rightsquigarrow$ Paramagnetic

Low T: $Q_{ab} = \left(\begin{array}{c|c} 1 & u \\ \hline u' & 1 \end{array} \right)_q \rightsquigarrow$ 1 step RSB

Today: We will study this model in the Euclidean time formalism. To any of you who have worked on SYK, this will seem very familiar.

Again we want to compute:

$\bar{F} = -\frac{1}{\beta} \int dJ P(J) \log Z$ with
 $Z = \int D\sigma \exp \left[- \int_0^\beta dt H \sigma_i(t) + \sum_{i_1 \dots i_p} J_{i_1 \dots i_p} \sigma_{i_1}(t) \dots \sigma_{i_p}(t) \right]$

This is the same model as SYK w/ 2 differences

1) σ_i are bosons: $\sigma_i(\tau+\beta) = \sigma_i(\tau)$ & the kinetic term is $\dot{\sigma}^2$

2) The spherical constraint $\sum_i \sigma_i^2 = N$
we will impose this by adding a term

$$\int_0^\beta d\tau z \left(\sum_i \sigma_i^2 - N \right)$$

As before we add Replicas to get rid of the logarithm

$$\bar{F}^n = \int \mathcal{D}\sigma \int dJ e^{-\frac{J_{i_1 \dots i_p}^2}{2} \frac{2N^{p-1}}{p!}} \exp \left\{ - \int_0^\beta d\tau \left[M \dot{\sigma}_i^a \dot{\sigma}_i^a + \sum_a z^a \left(\sum_i \sigma_i^2 - N \right) + J_{i_1 \dots i_p} \sigma_{i_1}^a \dots \sigma_{i_p}^a \right] \right\}$$

Integrate out J & integrate in Q_{ab} with

$$J = \int \mathcal{D}\lambda \mathcal{D}Q \exp \left\{ \int_0^\beta d\tau \int_0^\beta d\tau' \lambda_{ab}(\tau, \tau') \left(N Q_{ab}(\tau, \tau') - \sum_i \sigma_i^a(\tau) \sigma_i^b(\tau') \right) \right\}$$

$$\Rightarrow \bar{F}^n = \int \mathcal{D}\sigma \mathcal{D}\lambda \mathcal{D}Q \exp \left\{ - \int_0^\beta d\tau \int_0^\beta d\tau' \sigma_i^a(\tau) \left[\delta_{ab} \delta(\tau - \tau') \left[-M \dot{\sigma}_i^2 + z^a \right] + \lambda_{ab}(\tau, \tau') \right] \sigma_i^b(\tau') + N \int_0^\beta d\tau' \int_0^\beta d\tau \left(\lambda_{ab} Q_{ab} + \frac{1}{4} Q_{ab}(\tau, \tau')^p + \delta(\tau - \tau') \sum_a z^a \right) \right\}$$

→ This is the same as before but now with time dependence

Integrate out σ gives

$$\det^{-1/2} \left[2(\delta_{ab} \delta(\tau-\tau')) \{ -M \partial_{\tau}^2 + Z^* \} + \lambda_{ab}(\tau, \tau') \right]$$

or:

$$\begin{aligned} \bar{F}^N = & \int D\lambda DQ \exp \left\{ -\frac{N}{2} \log \det \left(2(\delta_{ab} \delta(\tau-\tau')) \{ -M \partial_{\tau}^2 + Z^* \} + \lambda_{ab} \right) \right\} \\ & + N \int_0^{\beta} d\tau \int_0^{\beta} d\tau' \left[\lambda_{ab}(\tau, \tau') Q_{ab}(\tau, \tau') + \frac{1}{4} Q_{ab}(\tau, \tau')^p + \delta(\tau, \tau') n Z(\tau) \right] \end{aligned}$$

What do we do now? \leadsto at large N use saddle

$$\delta \lambda_{ab}: Q_{ab}(\tau, \tau') - \left(2 \delta_{ab} \delta(\tau-\tau') \{ -M \partial_{\tau}^2 + Z^* \} + 2 \lambda_{ab} \right)^{-1} = 0$$

$$\Rightarrow \left\{ -M \partial_{\tau}^2 + Z(\tau) \right\} Q_{ab}(\tau, \tau') + \int_0^{\beta} d\tau'' \lambda_{ac}(\tau, \tau'') Q_{cb}(\tau'', \tau') = \frac{1}{2} \delta(\tau-\tau') \delta_{ab}$$

$$\delta Q_{ab}: \lambda_{ab}(\tau, \tau') = \frac{p-1}{4} Q_{ab}(\tau, \tau')^{p-1}$$

δZ : imposes spherical constraint $Q_{aa}(\tau, \tau') = 1 \forall a$

\leadsto By now this should look a lot like SYK

What's different? still have replica indices
we know RSB happens in the static case. SYK was designed to avoid spin glass behavior

