Interesting Saddles
Lecture 2
Quantum Glass

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__Recap from last time:__ Starting from a Hamiltonian

$$H = J_{i_1 \dots i_p} \sigma_{i_1} \dots \sigma_{i_p} \qquad \text{w/ } \sum_i \sigma_i^2 = N$$

$$\& \quad P(J_{i_1 \dots i_p}) \propto \exp\left\{ - \frac{J_{i_1 \dots i_p}^2}{2} \frac{2N^{p-1}}{p!} \right\}$$

Leads to saddle point equations:

$$\frac{\beta^2}{2} p \, Q_{ab}^{p-1} + (Q^{-1})_{ab} = 0 \qquad Q_{ab} = \frac{1}{N} \sum_{i=1}^{N} \langle \sigma_i^a \sigma_i^b \rangle$$

$a = 1, \dots N$

High T $\quad Q_{ab} = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \rightsquigarrow$ Paramagnetic

Low T: $\quad Q_{ab} = \begin{pmatrix} \begin{array}{cc|c} 1 & u & q \\ u & 1 & \\ \hline & \begin{array}{cc} 1 & u \\ u & 1 \end{array} & \\ q & & \ddots \end{array} \end{pmatrix} \overset{n}{\rightsquigarrow}$ 1 step RSB

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__Today:__ We will study this model in the Euclidean time formalism. To any of you who have worked on SYK, this will seem very familiar.

Again we want to compute:

$$\bar{F} = -\frac{1}{\beta} \int dJ \, P(J) \log Z \quad \text{with}$$

$$Z = \int D\sigma \exp - \int_0^\beta d\tau \, M \dot{\sigma}_i \dot{\sigma}_i + J_{i_1 \dots i_p} \sigma_{i_1}(\tau) \dots \sigma_{i_p}(\tau)$$

This is the same model as SYK w/ 2 differences

1) σ_i are bosons: $\sigma_i(\tau+\beta) = \sigma_i(\tau)$ & the Kinetic term is $\dot{\sigma}^2$

2) The spherical constraint $\sum_i \sigma_i^2 = N$

we will impose this by adding a term

$$\int_0^\beta d\tau\, z\left(\left\{\sum_i \sigma_i^2\right\} - N\right)$$

As before we add Replicas to get rid of the logarithm

$$\bar{F}^n = \int D\sigma \int dJ\, e^{-\frac{J_{i_1\ldots p}^2}{2}\frac{2N^{p-1}}{p!}} \exp\left\{-\int_0^\beta d\tau M \dot{\sigma}_i^a \dot{\sigma}_i^a + \sum_a Z^a\left\{\left(\sum_i \sigma_i^{a2}\right) - N\right\} + J_{i_1\ldots i_p}\sigma_{i_1}^a \ldots \sigma_{i_p}^a\right\}$$

Integrate out J & integrate in Q_{ab} with

$$1 = \int D\lambda\, DQ \exp\left\{\int_0^\beta d\tau' \int_0^\beta d\tau\, \lambda_{ab}(\tau,\tau')\left(N Q_{ab}(\tau,\tau') - \sum_i \sigma_i^a(\tau)\sigma^b(\tau')\right)\right\}$$

$$\Rightarrow \bar{F}^n = \int D\sigma\, D\lambda\, DQ \exp\left\{-\int_0^\beta d\tau \int_0^\beta d\tau'\, \sigma_i^a(\tau)\left[\delta_{ab}\delta(\tau'-\tau)\left[-M\partial_\tau^2 + z^a\right] + \lambda_{ab}(\tau,\tau')\right]\sigma_i^b(\tau') + N\int_0^\beta\int_0^\beta d\tau' d\tau\left(\lambda_{ab} Q_{ab} + \frac{1}{4} Q_{ab}(\tau,\tau')^p + \delta(\tau-\tau')\sum_a z^a\right)\right\}$$

→ This is the same as before but now with time dependence

Integrate out σ gives

$$\det{}^{-N/2}\left[2\left(\delta_{ab}\,\delta(\tau-\tau')\left\{-M\partial_\tau^2 + Z^a\right\} + \lambda_{ab}(\tau,\tau')\right)\right]$$

or:

$$F^n = \int D\lambda\, DQ\, \exp\left\{-\frac{N}{2}\log\det\left(2\left(\delta_{ab}\delta(\tau-\tau')\left\{-M\partial_\tau^2+Z^a\right\}+\lambda_{ab}\right)\right)\right.$$
$$\left. + N\int_0^\beta d\tau'\int_0^\beta d\tau\left[\lambda_{ab}(\tau,\tau')Q_{ab}(\tau,\tau') + \frac{1}{4}Q_{ab}(\tau,\tau')^p + \delta(\tau,\tau')\,n\,Z(\tau)\right]\right\}$$

What do we do now? $\rightsquigarrow$ at large N <u>use saddle</u>

$$\delta\lambda_{ab}:\quad Q_{ab}(\tau,\tau') - \left(2\,\delta_{ab}\,\delta(\tau-\tau')\left\{-M\partial_\tau^2+Z^a\right\}+2\lambda_{ab}\right)^{-1} = 0$$

$$\Rightarrow \boxed{\left\{-M\partial_\tau^2 + Z(\tau)\right\}Q_{ab}(\tau,\tau') + \int_0^\beta d\tau''\,\lambda_{ac}(\tau,\tau'')\,Q_{cb}(\tau'',\tau') = \frac{1}{2}\delta(\tau-\tau')\,\delta_{ab}}$$

$$\delta Q_{ab}:\quad \boxed{\lambda_{ab}(\tau,\tau') = \frac{p}{4}Q_{ab}(\tau,\tau')^{p-1}}$$

δZ: imposes spherical constraint $Q_{aa}(\tau,\tau) = 1$ $\forall a$

$\rightsquigarrow$ By now this should look <u>a lot</u> like SYK

What's different? still have replica indices

we know RSB happens in the static case. SYK was designed to avoid spin glass behavior

The Static approximation is the Same as before So we know the Solution should be 1-Step RSB

⇝ Proven in the Appendix of the referenced paper

<u>Interesting facts:</u> $Q_{ab}^{(\tau,\tau')} = \sum_i \langle \sigma_i^a(\tau)\, \sigma_i^b(\tau') \rangle$

So for $a \neq b$ this will be a product of 1 pt functions

→ 1 pt functions are independent of τ
So only Q_{aa} has time dependence!

2) Based on intuition from SYK, one would conclude

$$Q_{ab} = \frac{\delta_{ab}}{(\tau-\tau')^{\Delta}} \qquad \Delta = \frac{1}{p}$$

⇝ but because of RSB there is subtle interplay between diagonal and off diagonal terms which forces $\Delta = 2 \ \forall p$!

I have not verified this in detail (calculation is long) but it would be something fun to do over the weekend :)