
Black holes and the math that describes them

(I promise this won't hurt)

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Introduction

Why study black holes?

General Relativity

What is General Relativity?

Black Holes

How do we define a black hole?

Peculiarities of black holes

Quantum gravity

Conclusion

What have we learned?

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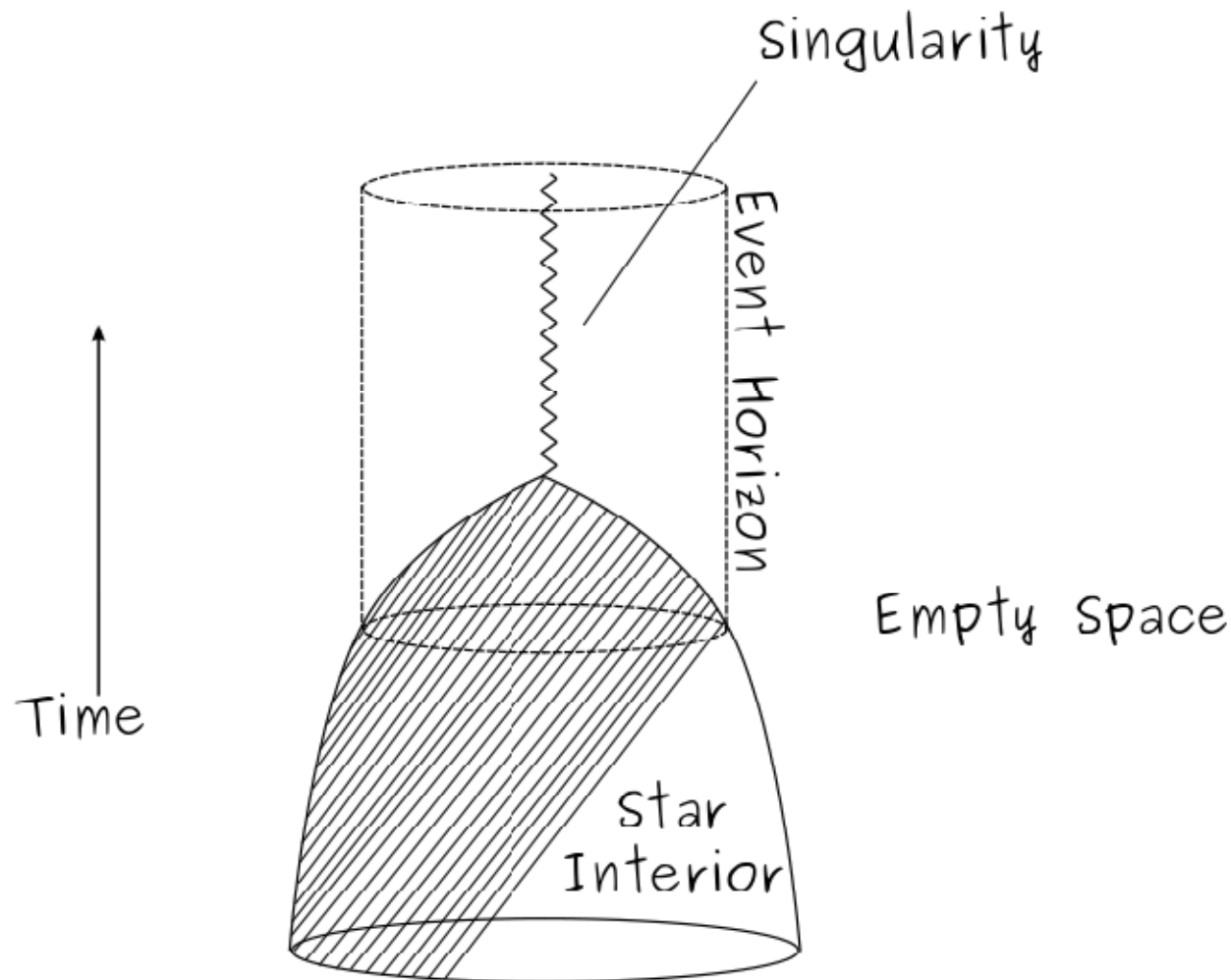
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Basic Question: What happens inside a black hole?



Black holes come from a star whose implosion can't be stopped by pressures that counteract the collapse.

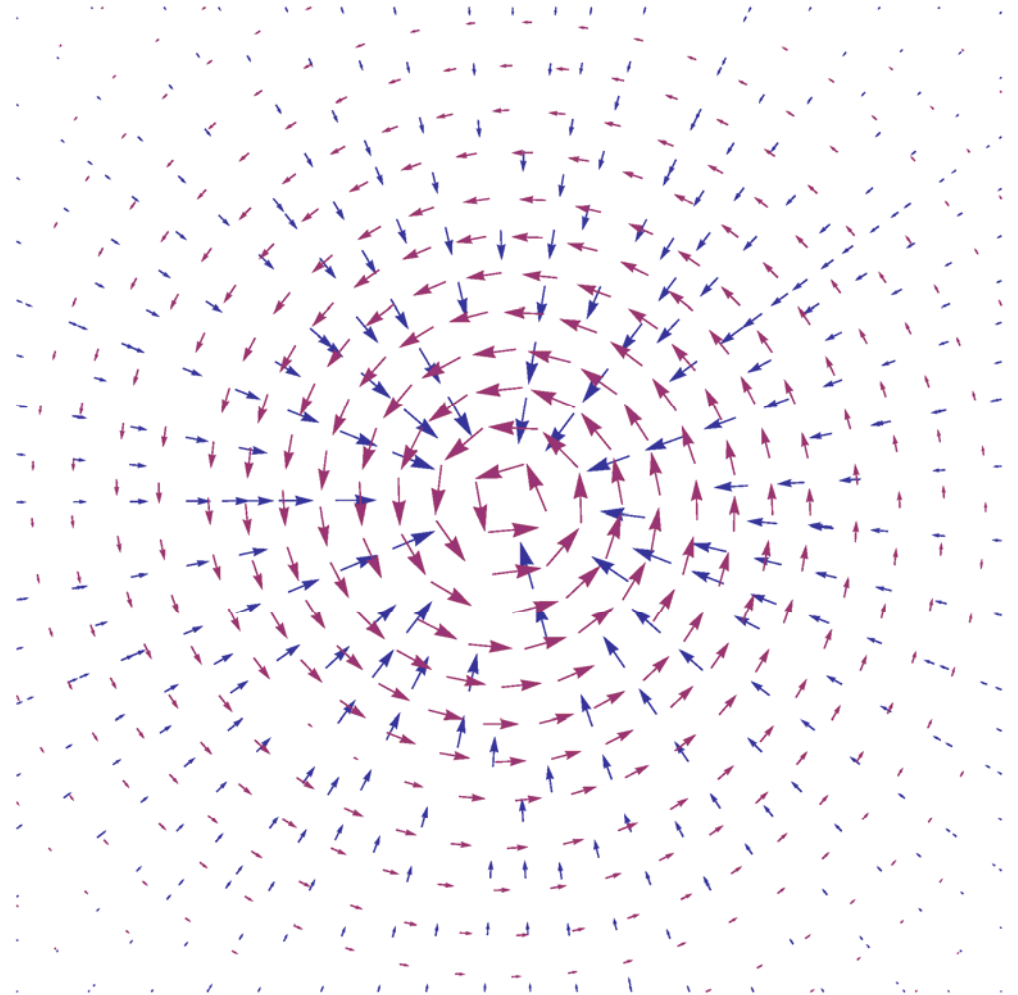
Singularity?

We encounter singularities a lot in physics.

Consider the attractive force of an electron on a positive charge.

Force goes as $1/\text{distance}^2$

Distance $\rightarrow 0$, force $\rightarrow \infty$

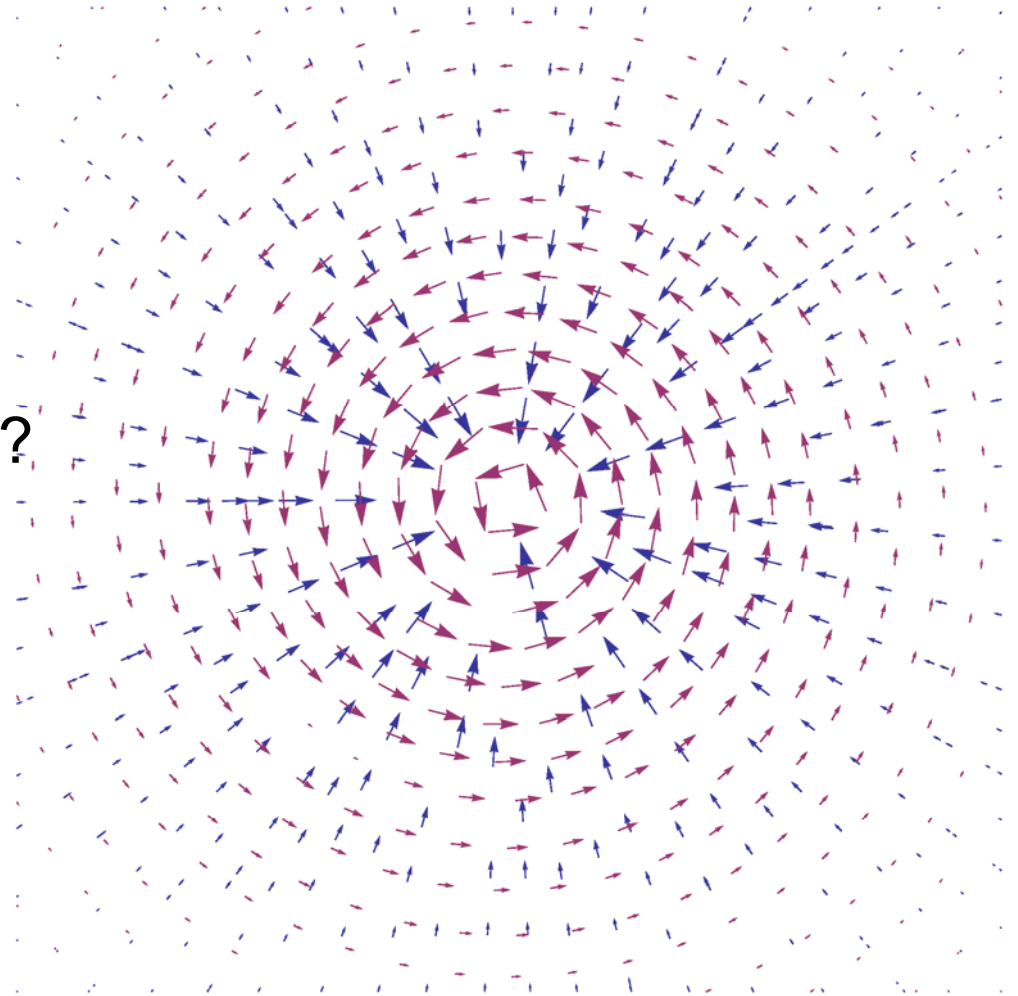


Singularity?

Infinities imply we have neglected some physics.

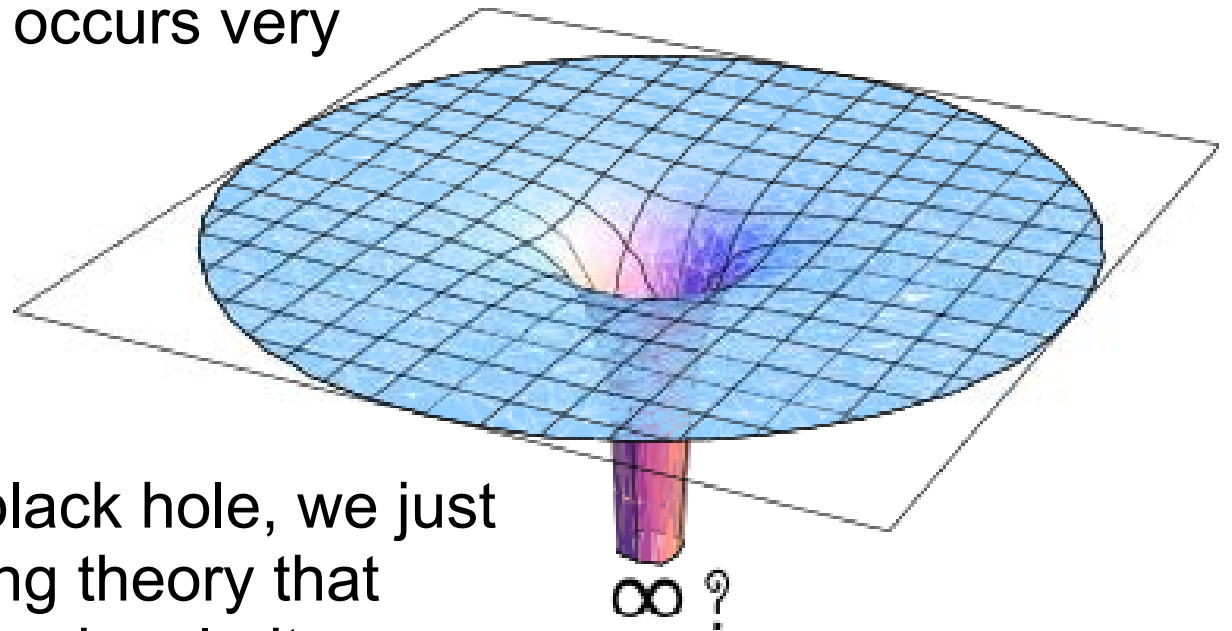
What have we forgotten here?

Quantum Mechanics!



Peculiarities of the black hole singularity?

Quantum mechanics fixes the nonsensical infinity that occurs very close to the electron.



Expect the same for a black hole, we just don't know the underlying theory that kicks in at the black hole singularity.

Quantum gravity

These questions have opened up the exciting field of research known as **quantum gravity**.

Theorists are trying to understand the fundamental structure of space and time at very small scales. Problem is, we have no experiments to lead us in the right direction!

Before delving into this deep subject, we had better understand the theory of black holes very carefully.

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What would Einstein say?

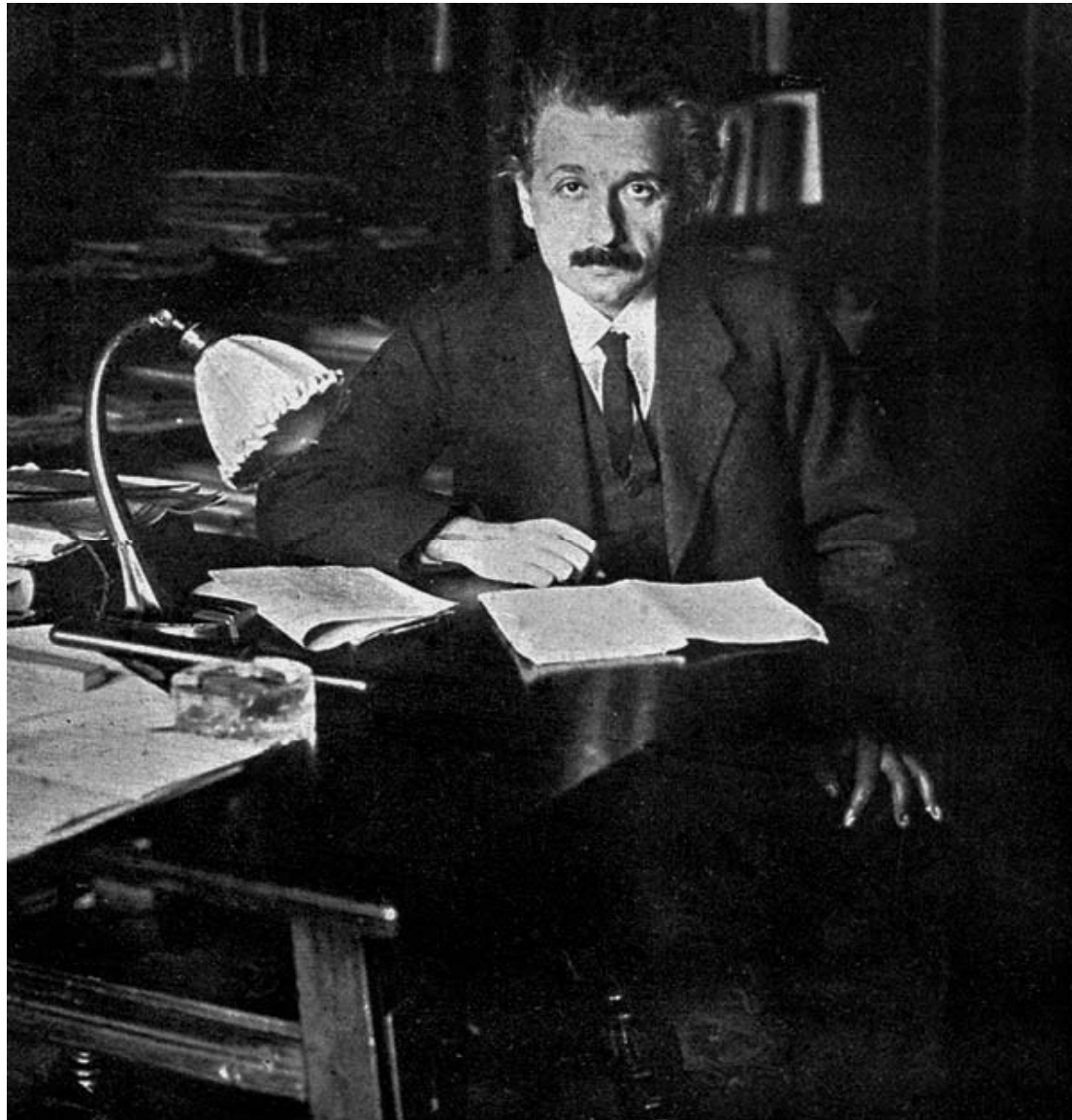


Image courtesy of wikipedia

Theory of rulers and clocks

We all recall from kindergarten that Einstein's equations are:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} .$$

What this really means is:

How one measures distance and time = Energy Density

Theory of rulers and clocks?

Einstein's equations measure how much matter and energy **bend** the geometry of space as well as how much they **slow down** clocks.

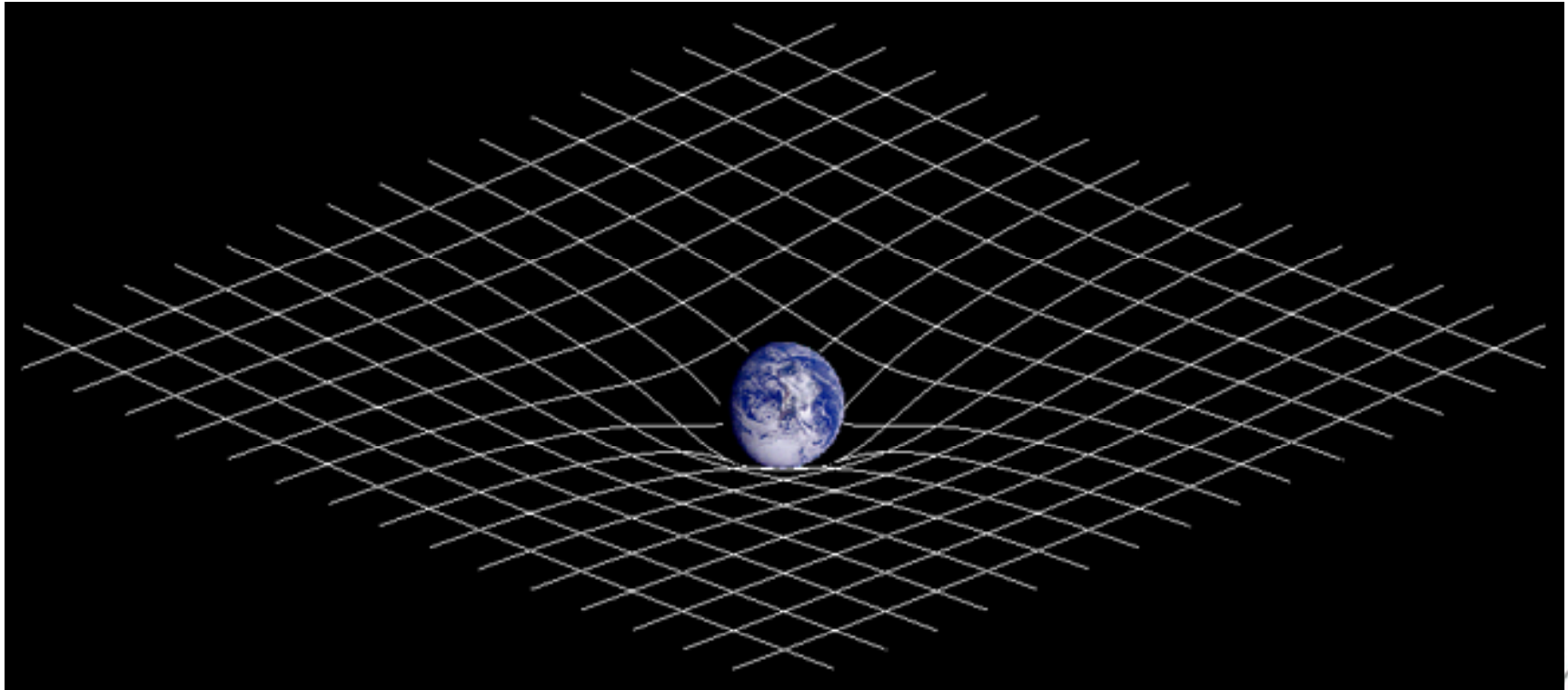


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Theory of rulers and clocks?

Slowing down of clocks can be viewed as a **bending of time**.

Both space and time being bent, more natural to combine the two into one object = **Spacetime!**

What is Spacetime?

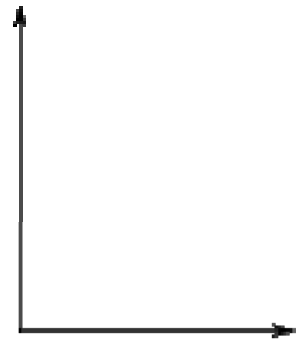
Spacetime is a geometric object, like a ball or a sheet, that contains time as one of its directions.



0d



1d



2d

What is Spacetime?

You can think of it like a flip book.

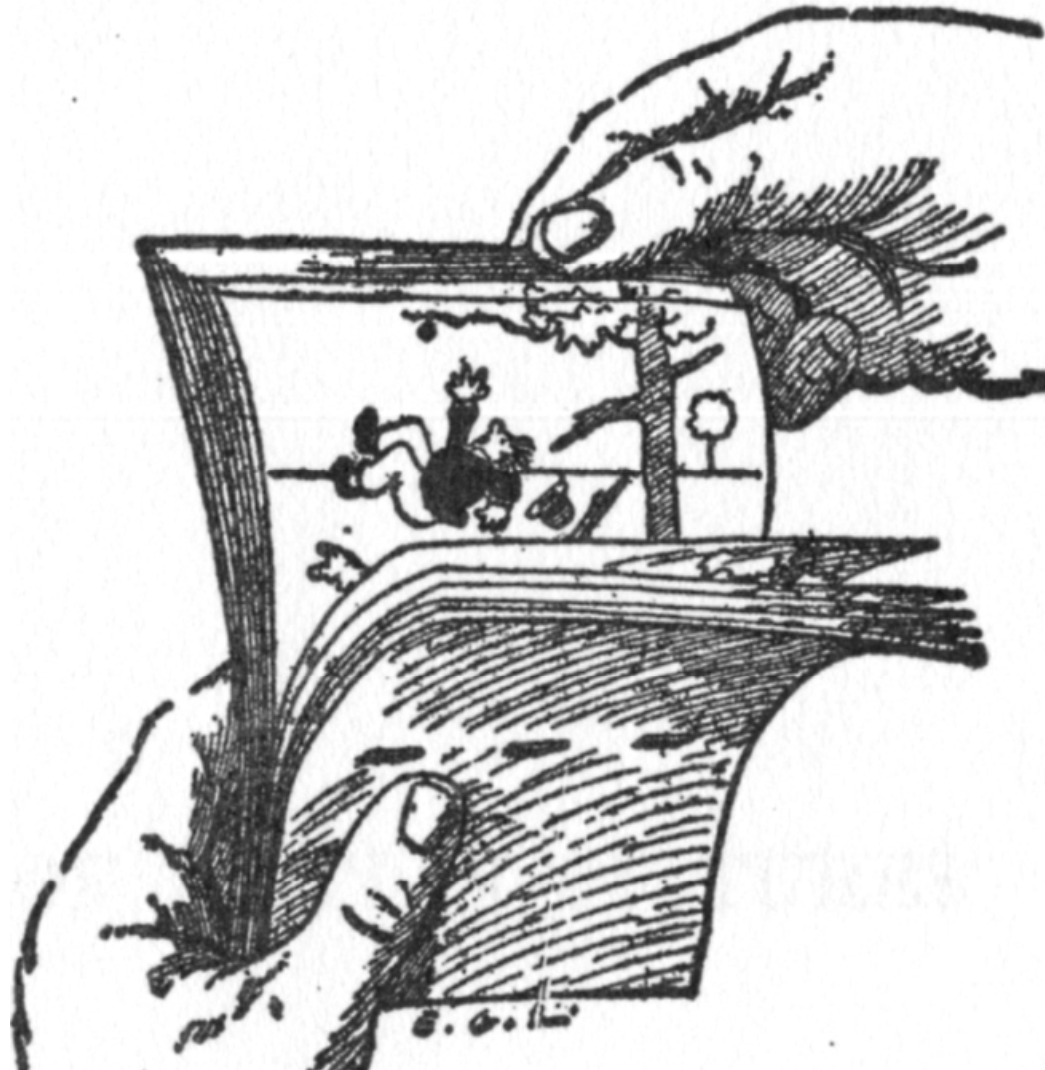
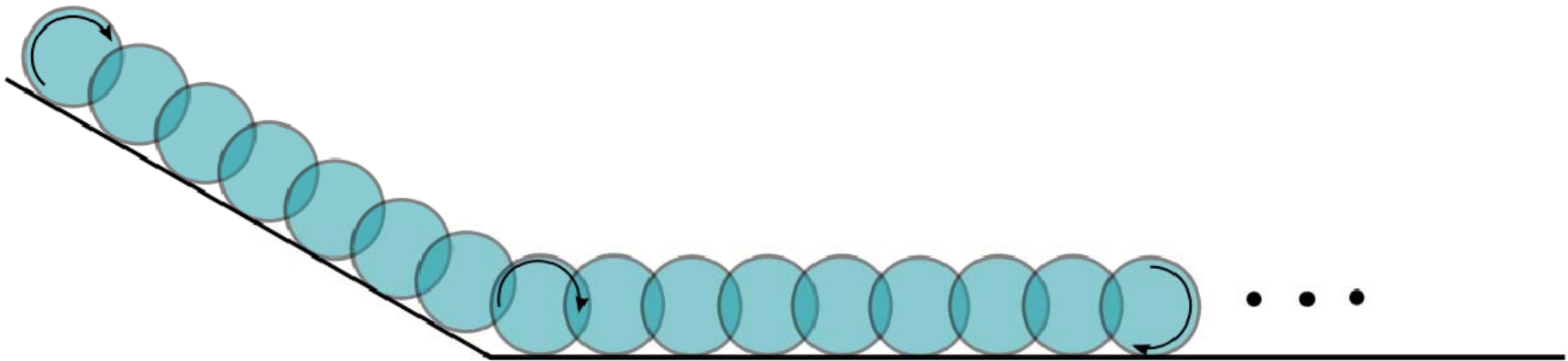


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What is Spacetime?



What is Spacetime?

Every page in the flip book represents a “**slice**” of spacetime along a certain choice of time coordinate.

Important: Solutions to Einstein’s equations give the structure of **all of space** and **all of time**.

We do not yet have a solution including all the sources of matter or else we would know the future of the universe!

What is the upshot of thinking about spacetime?

Spacetime encodes the structure of causality of the universe

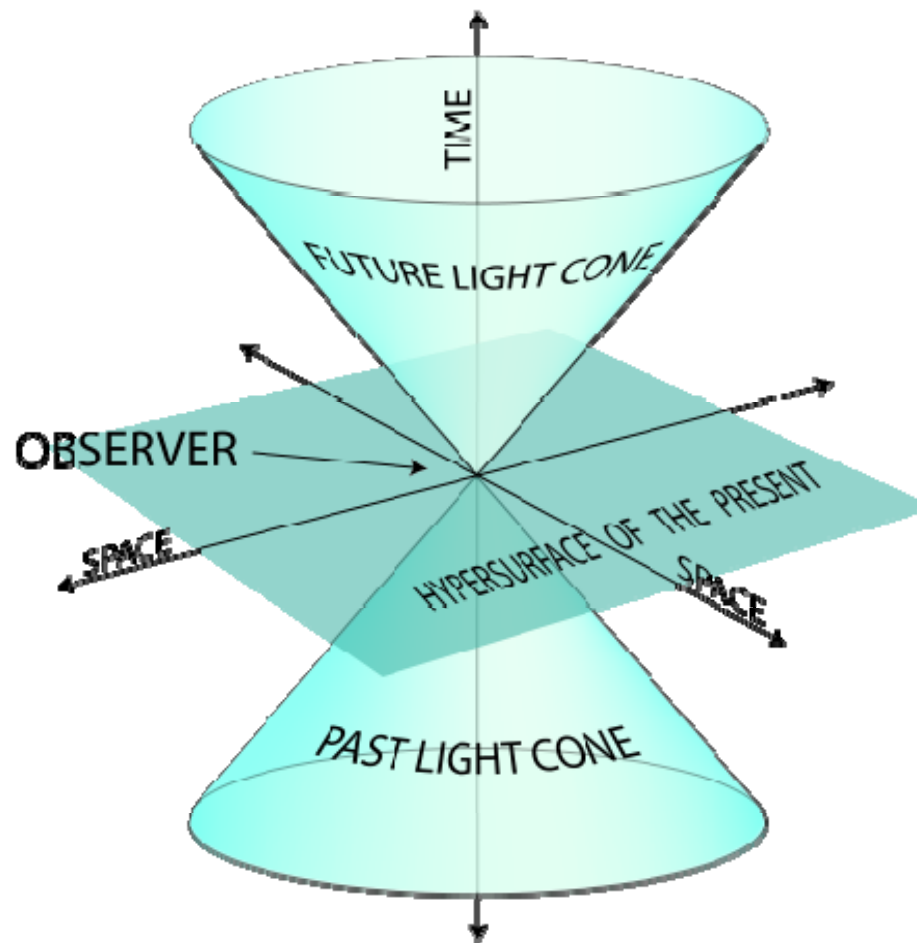


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How do we define a **black hole**?

Black holes have an event horizon ($v_{\text{escape}} > v_{\text{light}}$)

Define black hole as a spacetime with two completely causally disconnected regions of spacetime!

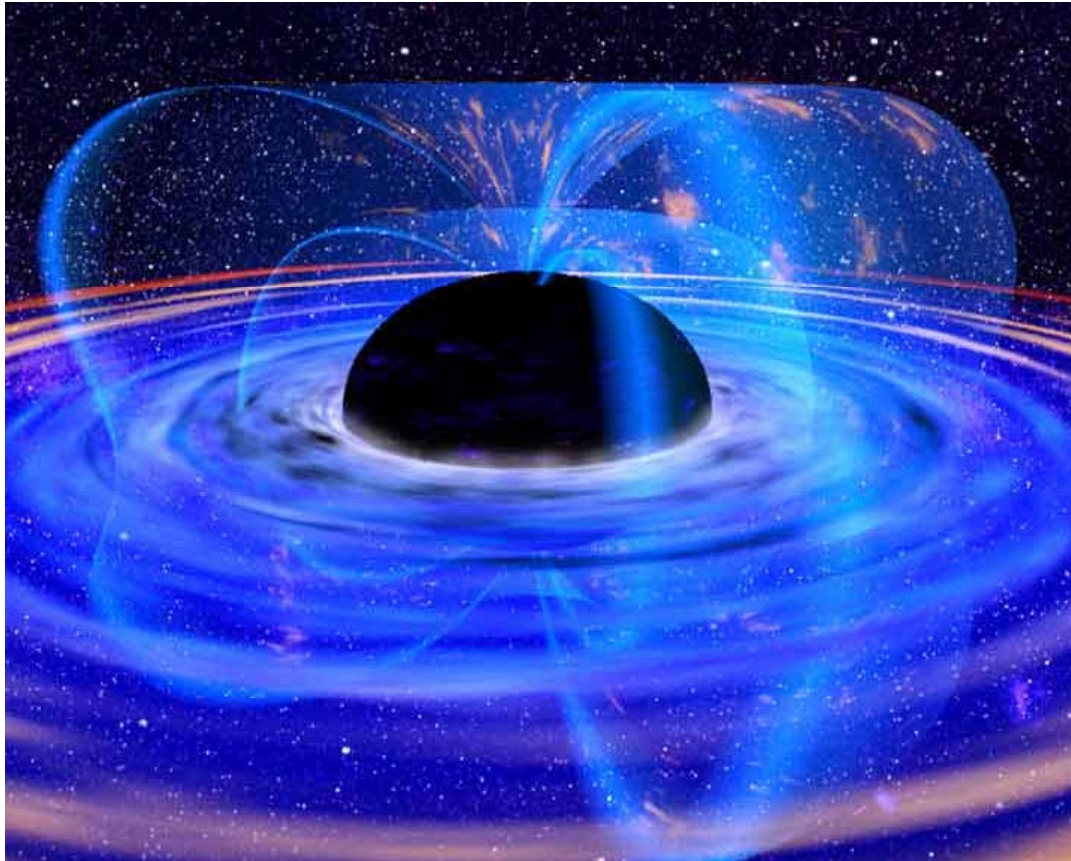


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What exactly is a **black hole**?

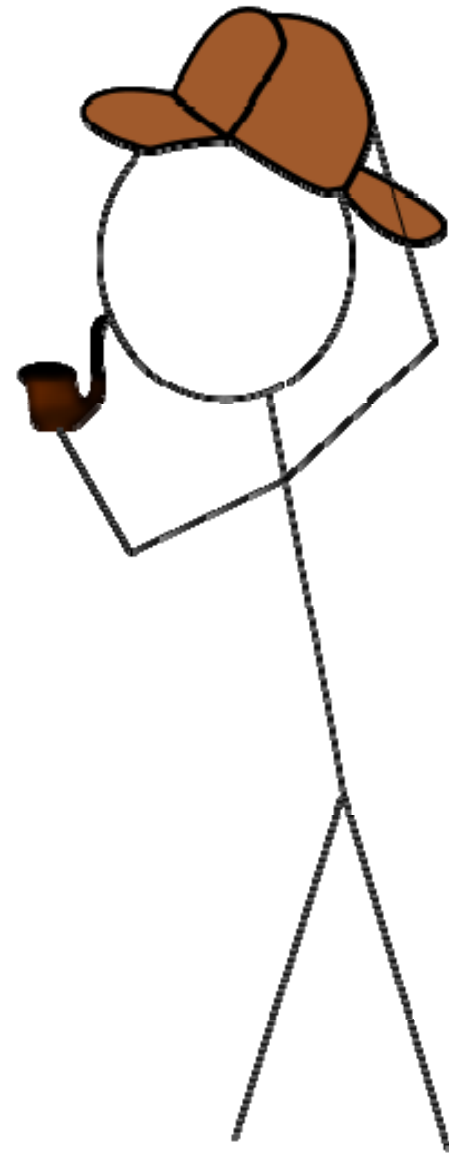
Actually that definition is not quite correct.

Event horizon = **one-way membrane**. We are in causal contact with stuff inside of black hole but **not vice-versa**.

Furthermore, we know of spacetimes with regions that are causally disconnected **which are not black holes**.

Mystery of the lost sandwich

Common view is that if you misplaced your sandwich and it somehow fell into a black hole, not even the best detective in the universe could find out what happened to it.



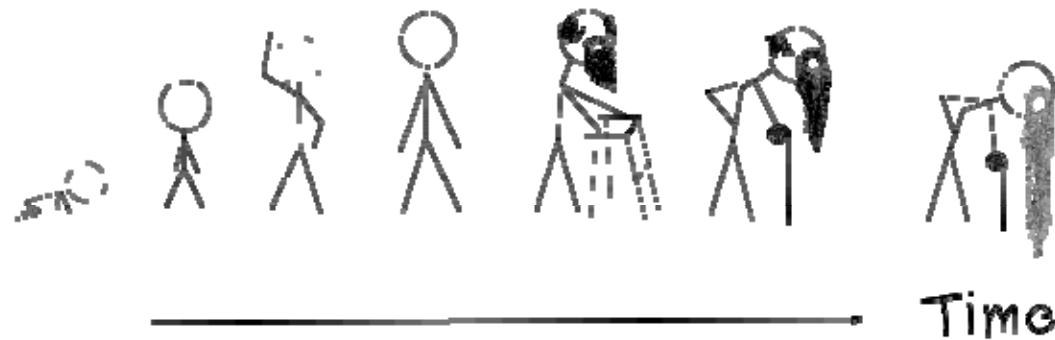
Mystery of the lost sandwich

What happened to the sandwich?

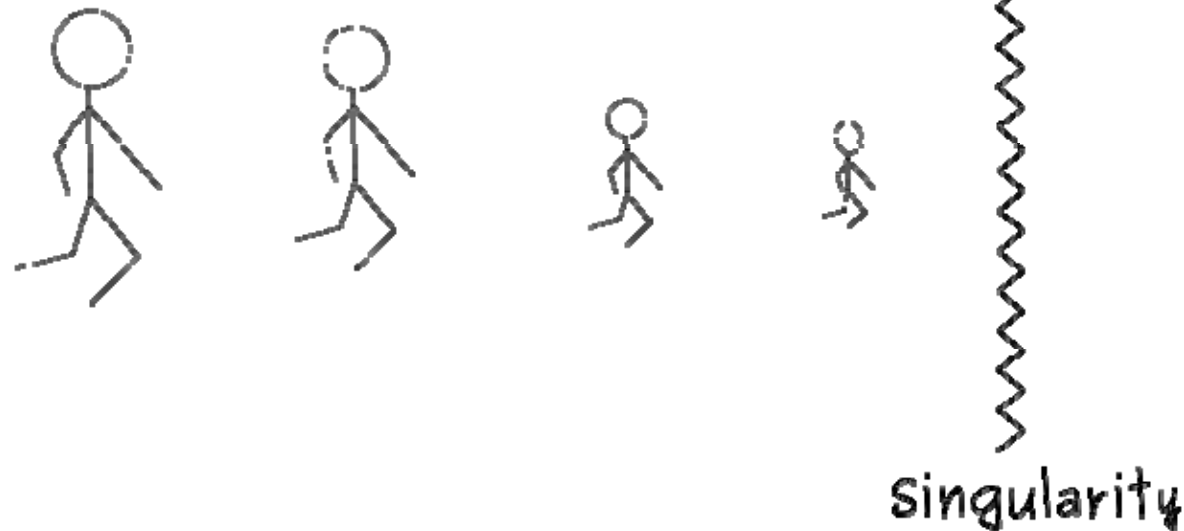
If we trust Einstein's equations: Sandwich fell into a region where time and space swap meaning.

Time and space swap meaning?

Outside a black hole

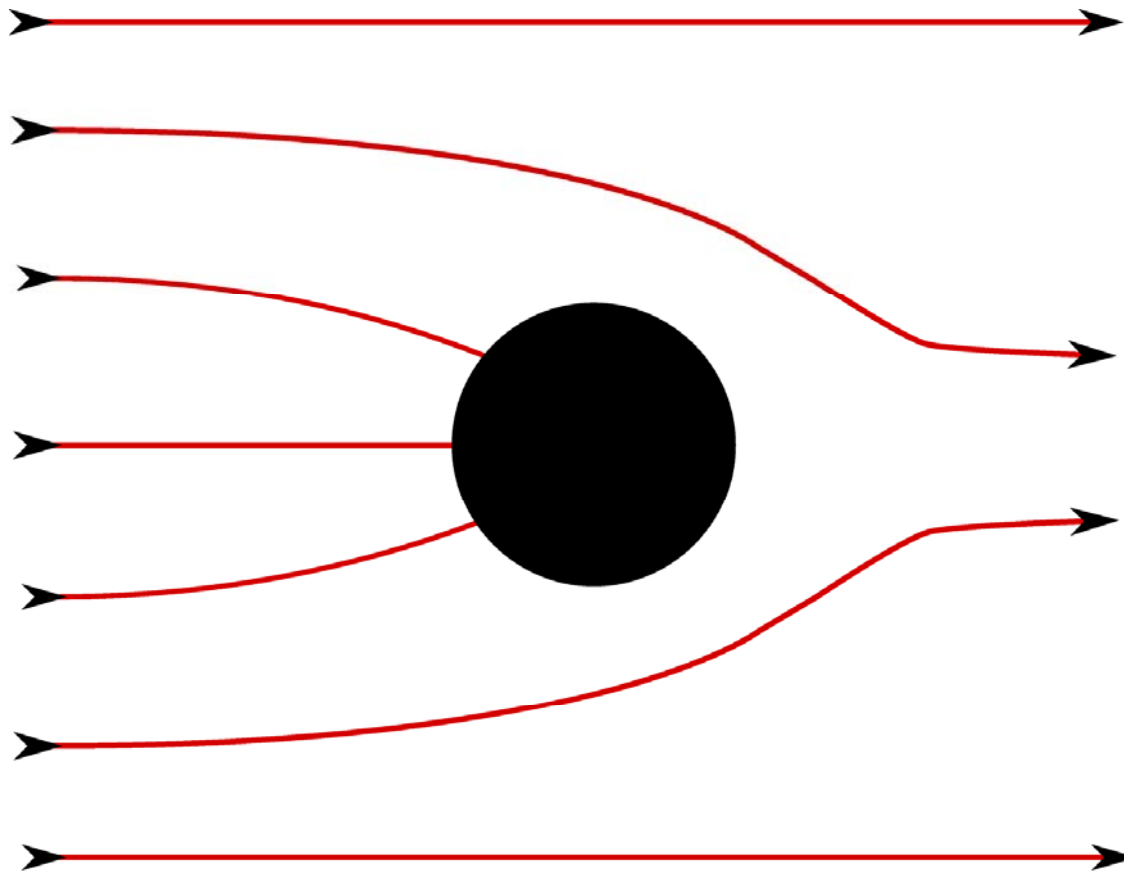


Inside a black hole



What exactly is a **black hole**?

Could define a black hole spacetime as one where: out of many sandwiches thrown, some end up in a region where time and space swap meaning and don't get infinitely far away.



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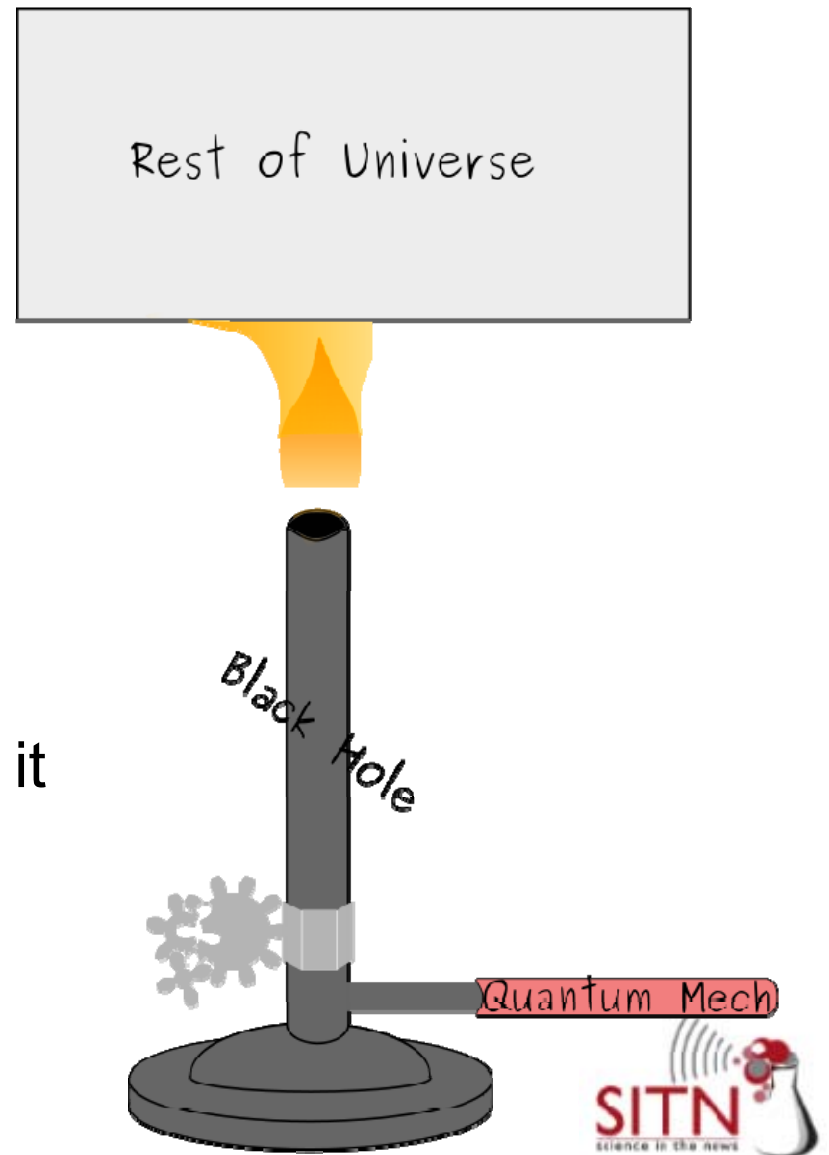
What have we learned?

Peculiarities of black holes

Actually this is again not quite right

Black holes radiate particles quantum mechanically! **Sandwich might pop out!**

A mathematical definition exists, but it is cumbersome to write down.



Peculiarities of black holes

Black holes leak energy!

Quantum gravity

By conservation of energy, this means that black holes evaporate, then disappear.

This is one of the most pertinent predictions of quantum gravity and any full theory of quantum gravity needs to include this feature.

Black holes act as a testing ground for our theories of quantum gravity!

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Solutions to Einstein's equations encode the entire history of the universe.

Black hole spacetimes have particles (sandwiches) that don't make it infinitely far.

The singularity implies something more fundamental is happening at small scales which we have yet to understand.

Quantum gravity predicts that thermal radiation can escape a black hole!

Thank you!

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