WHAT DOES A THEORETICAL PHYSICISTS DO ALL DAY—*really*?

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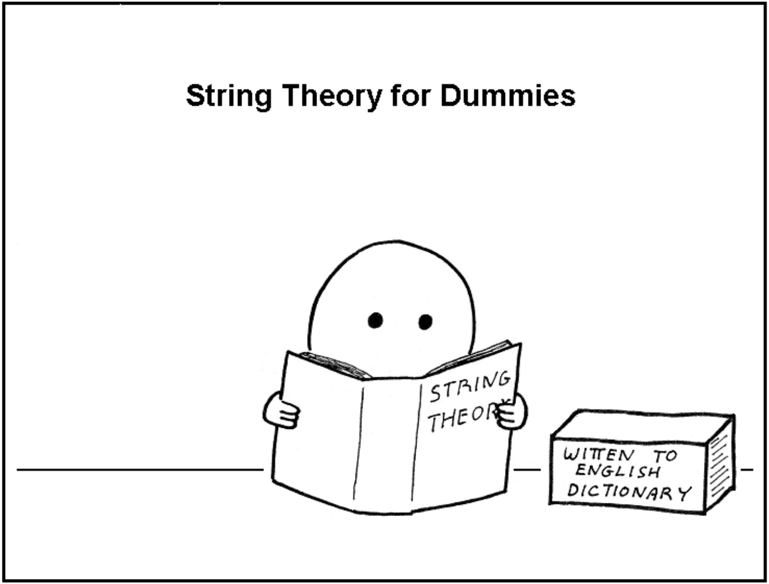
The typical ride when someone starts to learn ~~string theory~~ anything:



STEP 1: begin

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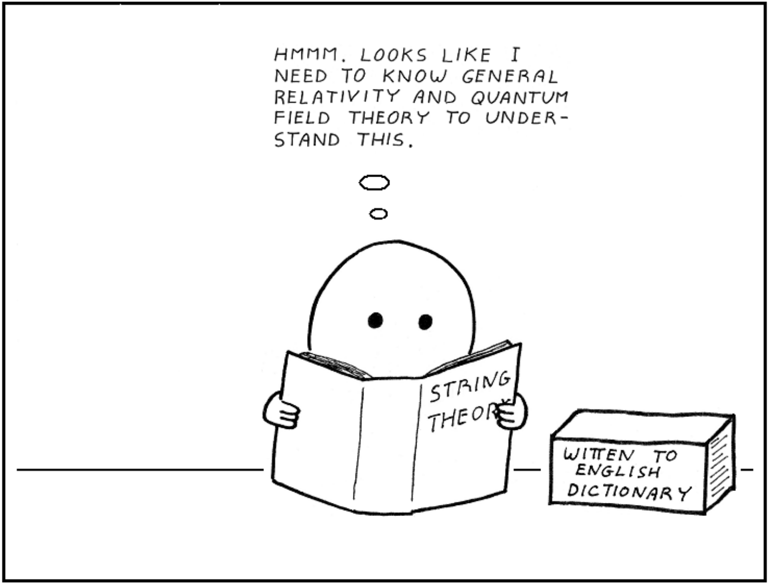

STEP 2: enter the rabbit hole

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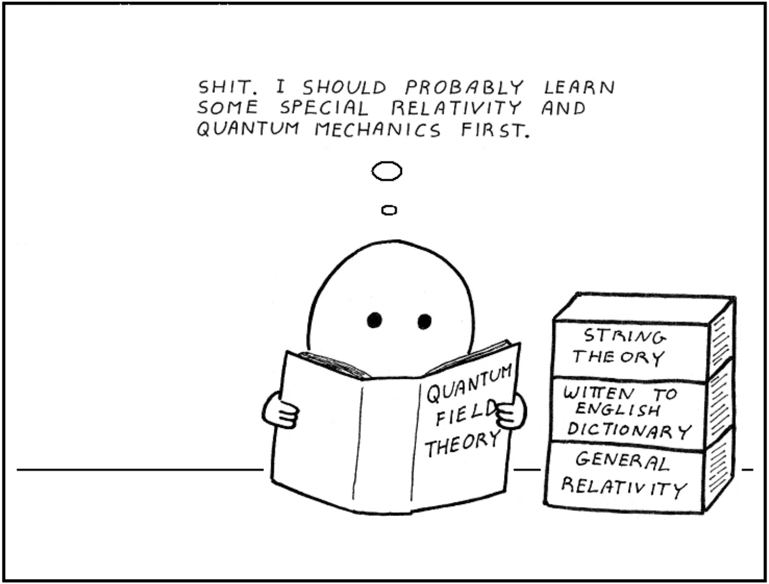

STEP 3: further down the rabbit hole

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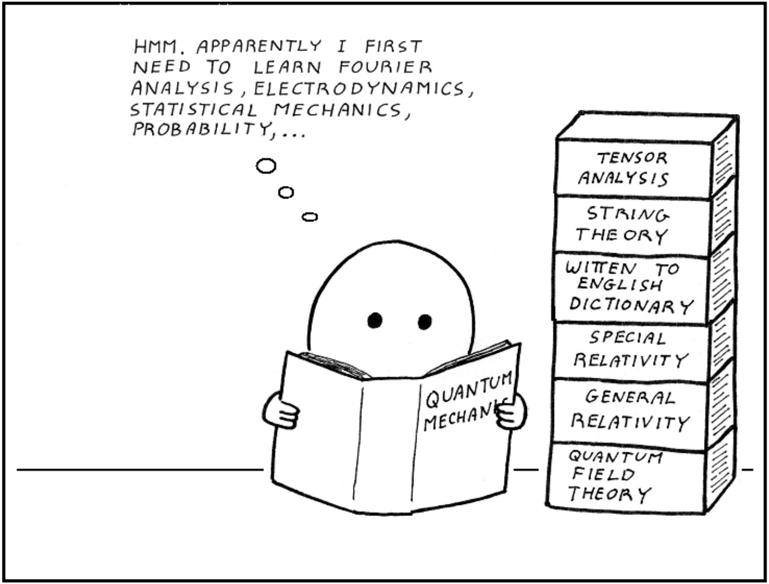

STEP 4: gradual realization

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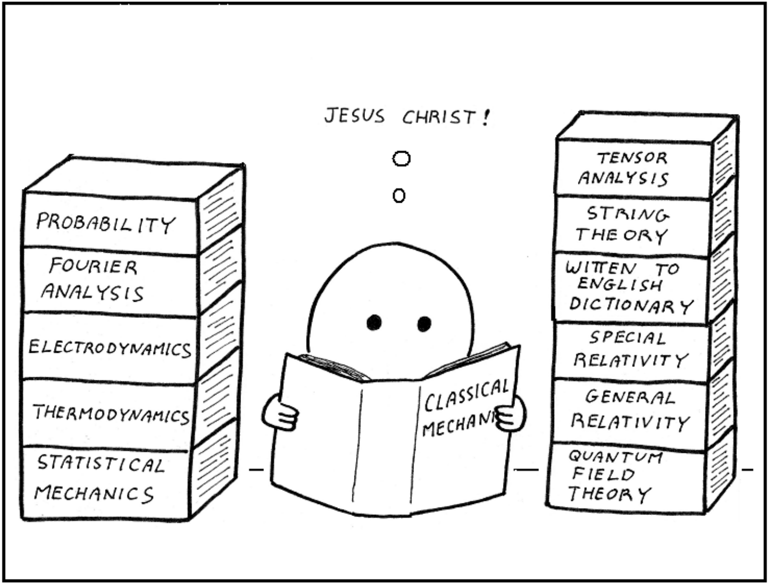

STEP 5: shock and awe

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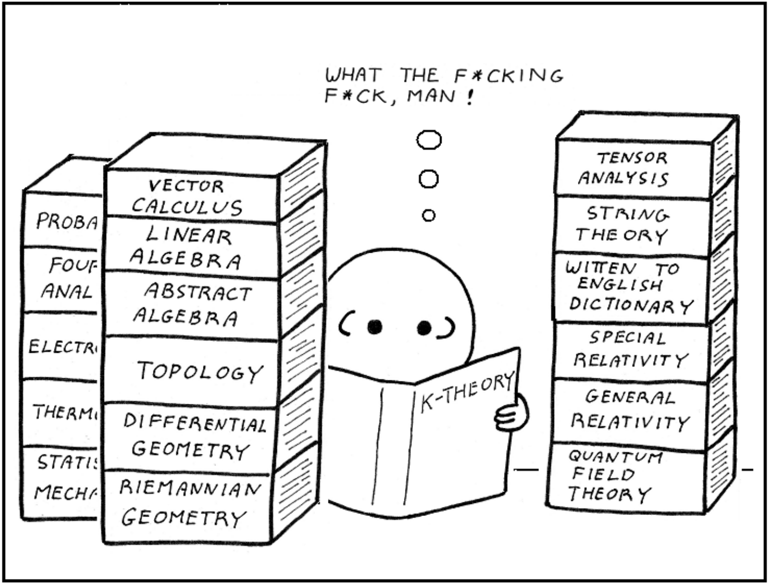

STEP 6: frustration

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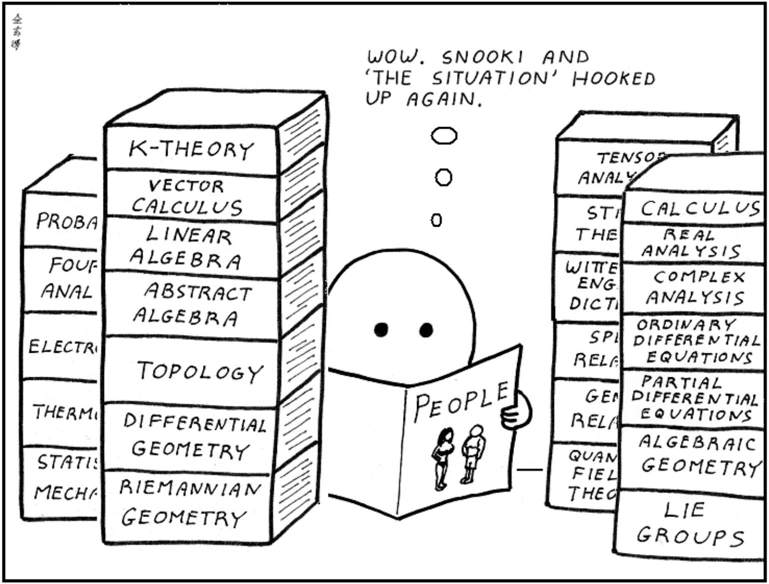

STEP 7: capitulation

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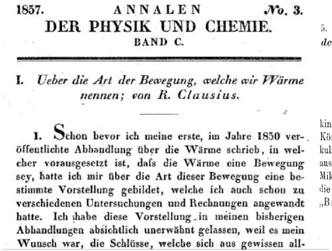
Instead of riding (and falling off) that roller coaster, my goal today will be to explain, using *simple* examples, how theoretical physicists go about their business.

Instead of riding (and falling off) that roller coaster, my goal today will be to explain, using *simple* examples, how theoretical physicists go about their business.

And instead of learning about the complicated details of black holes and string theory, let us start by going 150 years into the past.

1. Historical interlude
2. A solvable model
3. Black holes

150 years ago, physicists were obsessed with understanding the properties of gases



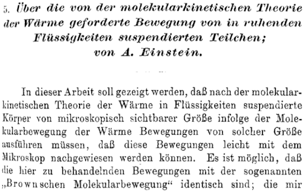
1857. ANNALEN №. 3.

DER PHYSIK UND CHEMIE.

BAND C.

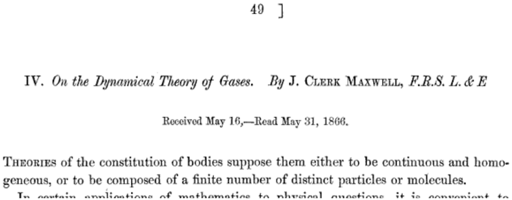
I. *Ueber die Art der Bewegung, welche wir Wärme nennen; von R. Clausius.*

1. Schon bevor ich meine erste, im Jahre 1850 veröffentlichte Abhandlung über die Wärme schrieb, in welcher vorausgesetzt ist, daſs die Wärme eine Bewegung sey, hatte ich mir über die Art dieser Bewegung eine bestimmte Vorstellung gebildet, welche ich auch schon zu verschiedenen Untersuchungen und Rechnungen angewandt hatte. Ich habe diese Vorstellung in meinen bisherigen Abhandlungen absichtlich unerwähnt gelassen, weil es mein Wunsch war, die Schlüsse, welche sich aus gewissen all-



5. *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen;* *von A. Einstein.*

In dieser Arbeit soll gezeigt werden, daß nach der molekularkinetischen Theorie der Wärme in Flüssigkeiten suspendierte Körper von mikroskopisch sichtbarer Größe infolge der Molekularbewegung der Wärme Bewegungen von solcher Größe ausführen müssen, daß diese Bewegungen leicht mit dem Mikroskop nachgewiesen werden können. Es ist möglich, daß die hier zu behandelnden Bewegungen mit der sogenannten „Brownschen Molekularbewegung" identisch sind; die mir



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IV. *On the Dynamical Theory of Gases.* *By* J. Clerk Maxwell, *F.R.S. L. & E*

Received May 16,—Read May 31, 1866.

Theories of the constitution of bodies suppose them either to be continuous and homogeneous, or to be composed of a finite number of distinct particles or molecules.

In certain applications of mathematics to physical questions, it is convenient to

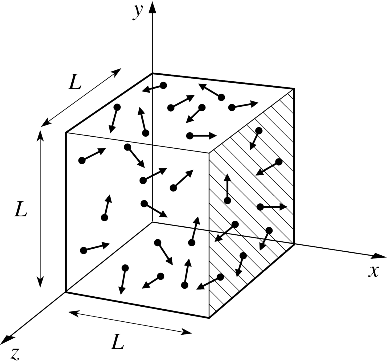
Experiments in the 1800s had revealed that most gases obey the *ideal gas law*:

$$\frac{PV}{T} = \text{constant} \; .$$

- P is the pressure
- V is the volume
- T is the temperature
- the constant is defined to be Boltzmann's constant k_B times the number of particles N

Physicists at the time wanted to understand this formula starting from basic principles.

Kinetic Theory



This led to the discovery of the the kinetic theory of gases in the 1850s:

its aim was to explain the *macroscopic* properties of a gas (pressure, volume, temperature, viscosity...) through the *microscopic* properties of the gas particles.

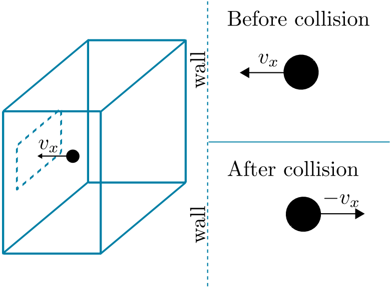
Kinetic Theory: basics

Let us consider the effect of a single gas particle on its container:[1]

Force on a wall by one particle:

$$F_{\text{single particle}} = \frac{\Delta p}{\Delta t}$$

- $\Delta p = 2mv_x$
- $\Delta t = 2 \times L/v_x$:
- $\boxed{F_{\text{single particle}} = \frac{mv_x^2}{L}}$



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[1] Clausius (1857).

Kinetic Theory: basics

The total force on the wall comes from accounting for all the N particles

$$F_{\text{total}} = N \times \frac{m\langle v_x^2\rangle}{L}$$

and $\langle v_x^2\rangle$ is the *average* velocity in the x direction. The total average velocity is :

$$\langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle$$

so if we assume that the average velocities in all directions are the same then we find:

$$\boxed{F_{\text{total}} = \frac{1}{3}N \times \frac{m\langle v^2\rangle}{L}}$$

Kinetic Theory: basics

Now the pressure on the wall is the total force per unit area:

$$\begin{aligned} P &= \frac{F_{\text{total}}}{L^2} = \left(\frac{1}{3}N \times \frac{m\langle v^2\rangle}{L}\right)/L^2 \\ PV &= \frac{1}{3}Nm\langle v^2\rangle \end{aligned}$$

where the volume $V = L^3$. So we've established

$$\boxed{PV \propto N \times \langle \text{kinetic energy per particle} \rangle}$$

Experiments

But we never measure the average kinetic energy of each gas particle in experiments!

This is why experimental scientists hate theoretical scientists.

Remember, our goal was to understand the origin of the *ideal gas law*

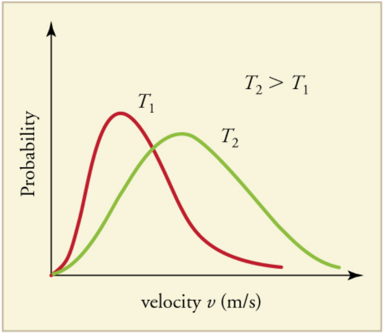
$$PV = Nk_BT \ .$$

So if we trust our theoretical derivation, it implies:

$$\boxed{\langle v^2 \rangle = 3k_BT/m}$$

Maxwell hits the scene

In 1866 James-Clerk Maxwell realized this equation $\langle v^2 \rangle = 3k_BT/m$ implies that particle velocities follow a probability distribution



$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{m}{2k_BT}\left(v_x^2+v_y^2+v_z^2\right)}$$

Thinking about the problem probabilistically was the key that led to many more derivations including heat conductivities and viscosity.

Brief Pause

Let's recap. We:

1. calculated the force of a single particle on the wall
2. derived a relation between PV and $\langle v^2 \rangle$
3. compared our results to experiment where $PV = Nk_BT$
4. concluded that the distribution of particle velocities depends on the system temperature.

But reality was much messier than that. We have the benefit of hindsight to tell a nice story, but for years physicists argued as to whether the kinetic theory was useful for anything at all.

Statistical mechanics

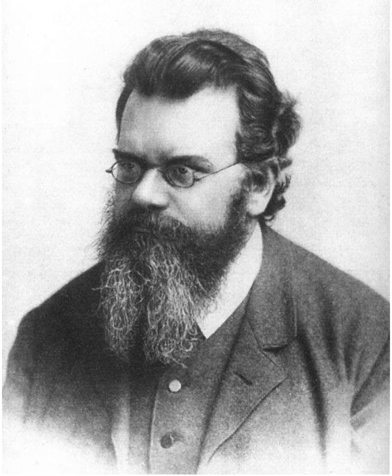
There are a few problems with the kinetic theory and Maxwell's distribution:

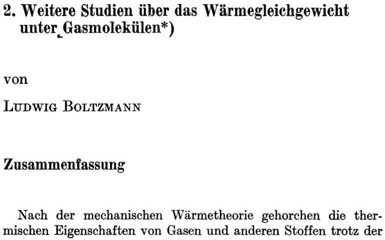
1. Not all gases obey the ideal gas law.
2. Some gases can be ideal, but mixtures of two species of particles, what then?
3. What real lessons do we learn?

The job of the theorists of the time was to explain how gases work *in general*. How do we generalize the lessons of ideal gases to other systems?

Statistical Mechanics

In 1872 Ludwig Boltzmann burst onto the scene





2. Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen*)

von

Ludwig Boltzmann

Zusammenfassung

Nach der mechanischen Wärmetheorie gehorchen die thermischen Eigenschaften von Gasen und anderen Stoffen trotz der

Statistical Mechanics

By simply looking at Maxwell's distribution:[2]

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{m}{2k_B T}\left(v_x^2+v_y^2+v_z^2\right)}$$

Boltzmann noticed that it was of the form:

$$f = \mathcal{N} e^{-\beta \times E}$$

- $\beta = \frac{1}{k_B T}$
- E is the *kinetic* energy.

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[2] now called Maxwell-Boltzmann

Statistical Mechanics

The interpretation of this equation is that the probability of our gas being in any one state is given by

$$p_{\text{state}} \propto e^{-\beta E_{\text{state}}} \qquad\qquad \beta = \frac{1}{k_B T}$$

Statistical Mechanics

Of course, we need probabilities to add up to 1:

$$\sum_{\text{states}} p_{\text{state}} = 1 \ ,$$

so the probability is really

$$p_{\text{state}} = \frac{e^{-\beta E_{\text{state}}}}{Z} \qquad\qquad Z = \sum_{\text{states}} e^{-\beta E_{\text{state}}}$$

For the case of the ideal gas

$$E_{\text{state}} = \sum_{\text{particles}} \frac{1}{2} m \vec{v}^{2}_{\text{particle}}$$

is the kinetic energy of all the particles. But what if E were something different? For example, what if E took into account interparticle interactions?

Statistical Mechanics

Boltzmann's first insight was that E is system dependent. It could include kinetic, potential and interatomic interactions.

If we want to describe a gas that *doesn't* obey the ideal gas law, we need to generalize the energy function E.

Boltzmann's second insight was that most equilibrium thermodynamic properties of a system could be calculated from Z, often called the partition function:

$$Z(\beta) = \sum_{\text{states}} e^{-\beta E_{\text{state}}}$$

For example the free energy $F = -\beta^{-1} \ln Z(\beta)$.

Nowadays, the way we associate an energy to a particular state of our collection of particles is what we call *a theory*.[3] A commonly used synonym for this term is *a model*.

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[3] This is true of the theory of electromagnetism or general relativity.

Statistical Mechanics

Boltzmann sadly did not live to see these insights fully appreciated. In 1906 he committed suicide, and some believe it was because his peers ridiculed him for these concepts.

By now, Boltzmann's formulation of thermodynamics has passed every conceivable experimental test.

Statistical Mechanics, what's the point?

One important point to take from all of this is that the partition function

$$Z(\beta) = \sum_{\text{states}} e^{-\beta E_{\text{state}}}$$

(which was derived to understand gases) can be used to study *any system* where there is a function that assigns a state → energy.

This concept is used in every subfield of theoretical physics, from the study of gases to string theory! It is also at the origin of quantum mechanics, although I won't get into that.

Statistical Mechanics, what's the point?

The downside is that, in practice, the partition function

$$Z(\beta) = \sum_{\text{states}} e^{-\beta E_{\text{state}}}$$

is very difficult if not impossible to compute.

If I were to summarize a large chunk of theoretical physics work it would be:

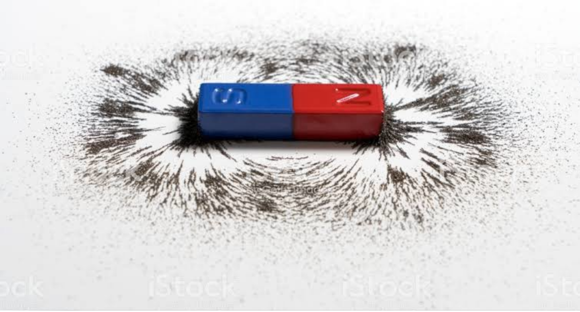
1. The search for *models* where $Z(\beta)$ may be computable
2. Trying to compute $Z(\beta)$
    2.1 If successful, extract physical insights

1. Historical interlude
2. A solvable model
3. Black holes

I've claimed that we can study the partition function of any system, not just gases. So now I'm going to try and convince you that we can use the partition function to study magnets.

What do we know about magnets?

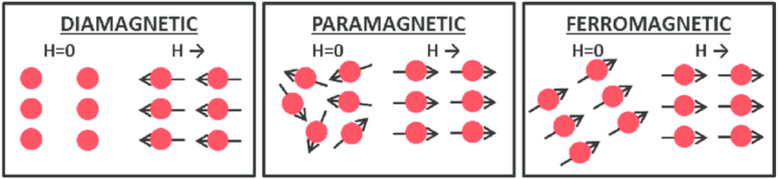
1. At home, you probably have magnets on your fridge. These have a magnetic dipole.
2. What you might not know, is that if you heat up a magnet, it loses this magnetic dipole.



What do we know about magnets?

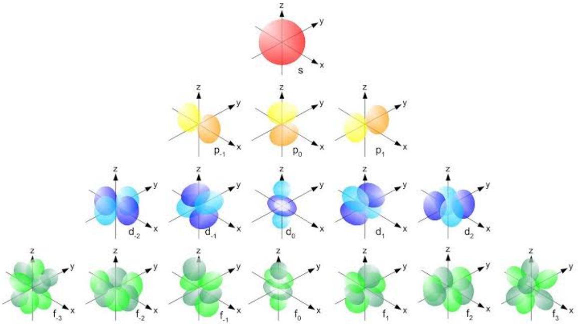
This is true of all bar magnets! It *doesn't matter* what they're made of.

Let us try to build a model of this phenomenon. To be more specific, the model should explain the property that bar magnets have nonzero magnetization in the absence of an external magnetic field— a.k.a *ferromagnetism*



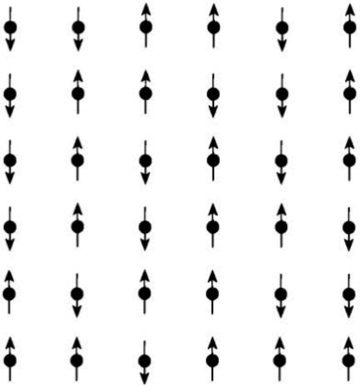
And the model should explain the property that if you heat up the magnet, it loses its magnetism.

The natural place to start building such a model would be to appeal to atomic physics and orbitals etc.



But that's too complicated!

Lenz 1920:

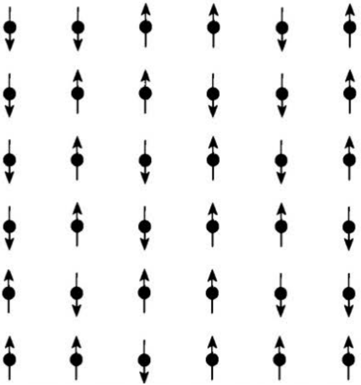


We will simply assume that we have a lattice of immovable spins $\sigma = \{\pm 1\}$ and

$$E = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - H \sum_i \sigma_i$$

- the total energy is lowered by J whenever two spins are aligned
- H is an external magnetic field, if a spin points up, E is lowered by an amount H.

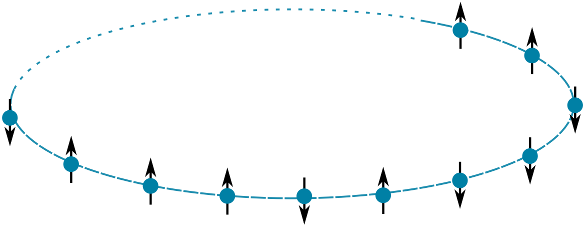
This is a *gross* approximation of how a magnet should work. The surprise is that it makes extremely precise predictions.



$$E = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - H \sum_i \sigma_i$$

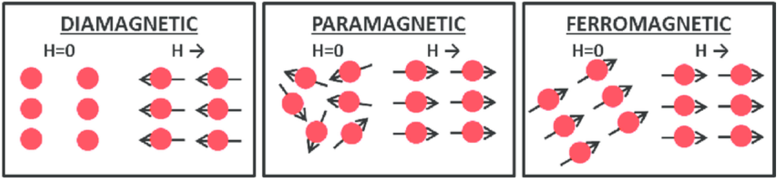
- $Z(\beta, H)$ for the 1d lattice was obtained by Ising in 1925
- $Z(\beta, H = 0)$ in 2d was solved by Onsager in 1944
- The solution for the 3d model is still not known.

Solution in 1d



Unfortunately the magnetization $M(H,T)$ of the system for a 1d chain of spins does not display ferromagentism.

$$M(H=0,T)=0$$



1d chains are paramagnets.

$d = 1$ chain

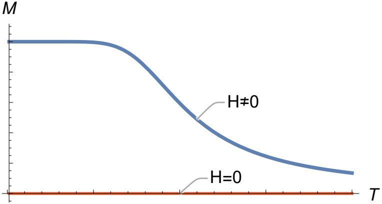
$$Z = \left[\cosh \beta H + \sqrt{\sinh^2 \beta H + e^{-4\beta J}}\right]^{\beta J N}$$

Try calculating these at home

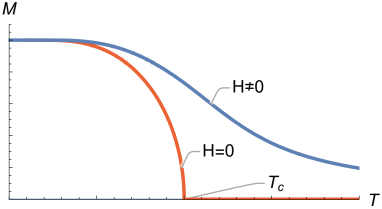
$d \to \infty$ lattice

$$Z = e^{-2\beta J M^2} \left[2 \cosh \left\{\beta \left(H + 4MJ\right)\right\}\right]^N$$
$$M = \tanh \left\{\beta \left(H + 4MJ\right)\right\}$$

$d = 1$ chain



$d > 1$ lattice



These plots are obtained by studying the partition function $Z(\beta, H)$ of this system of immovable spins.

T_c is the phase transition temperature.

This model can be used to study many more properties of magnets, such as its specific heat or magnetic susceptibility, or heat conductivity. And it gets many things right!

Amazing because we know that magnets are made of much more complicated things than these ± 1 bound spins.

1. Historical interlude
2. A solvable model
3. Black holes

My job is to understand black holes.

That means I want to come up with a model *as dumb* as the model where a magnet is built out of a bunch of ± 1 spins.

Why is that hard?

Remember how we built a model of a magnet:

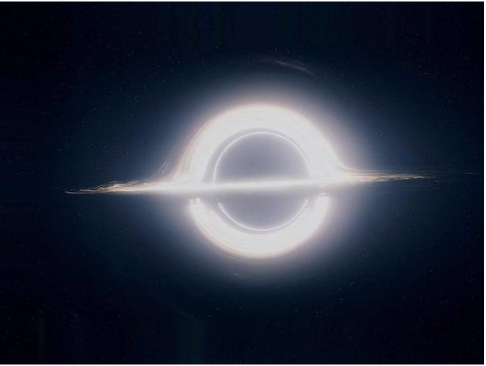
1. Do experiments on magnet to determine its properties
2. Guess at a model that reproduces these properties
3. Calculate!
4. Compare calculations with experiments

Why is that hard?

Let's try to do this for a black hole, treating it like a material or a state of matter.

1. ~~Do experiments on BH to determine its properties~~
2. ~~Guess at a model that reproduces these properties~~
3. Calculate!
4. ~~Compare calculations with experiments~~
5. Use calculations to determine what we expect of black holes
6. Are there any materials on earth that exhibit these properties?

What have we calculated about BHs?



- They have *negative* specific heat: if you *add* energy, T decreases
- The entropy scales with the *area* instead of the *volume*
- To best approximation: they radiate with an exact black body spectrum

No material on earth has all of these properties

Scientists have worked for over 40 years to come up with something that may model these phenomena, but it will take the duration of an entire PhD for me to tell you what those are. So I will stop here.

Lesson

I just want you to takeaway that we theorists study black holes using the *exact* same tools we use to study magnets.

THANK YOU